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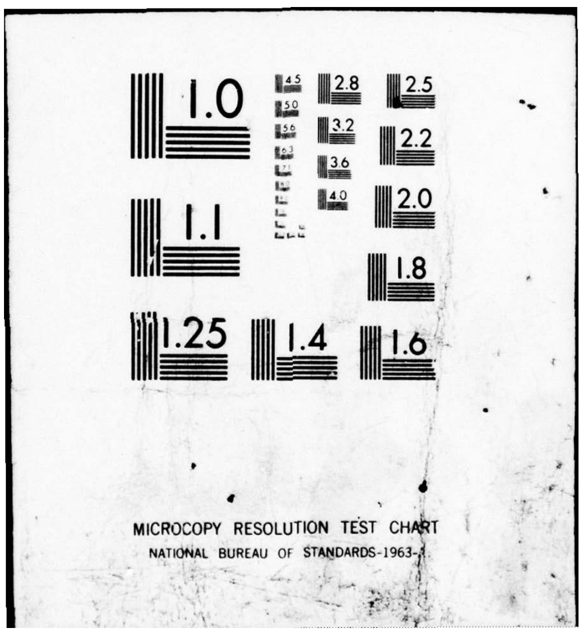
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RELATIONSHIP BETWEEN ACCEPTANCE-TEST RELIABILITY AND OPERATIONAL RELIABILITY

15 November 1966

Prepared for
U. S. NAVAL AMMUNITION DEPOT
CRANE, INDIANA
under Contract N164-11329

ARINC RESEARCH CORPORATION

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By

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Annapolis, Maryland 21401
Publication 538-01-1-696

SUMMARY

This report describes an investigation of the relationships between equipment reliability estimates as measured during manufacturer's reliability demonstration testing and as measured under operational conditions. The work was performed by ARINC Research Corporation for the Naval Ammunition Depot, Crane, Indiana

One of the earliest comprehensive studies of reliability was performed by the Advisory Group on Reliability of Electronic Equipment (AGREE). Task Group 3 was charged with the development of "basic requirements for tests . . . which will prove conclusively that the equipment will meet the minimum acceptable figure for reliability established for the equipment type".

The study documented in this report indicated that there is a definite relationship between airborne equipment reliability estimates as measured during AGREE testing and as measured under operational conditions. This is the first time that such a relationship has been established by data from as many different equipments (18) as were used in this study.

The results indicate -- with a data correlation of 0.81 -- that the relationship between operational MTBF and AGREE-test MTBF may not be 1-to-1, and is expressed as:

$$\theta_o = 0.017\theta_A^{\ 1.76}$$

where

θ_o = MTBF measured under operational conditions

θ_A = MTBF measured under AGREE-test conditions

This relationship represents equipments whose MTBF values ranged from 12 to 301 hours during operational use and from 47 to 454 hours during AGREE testing.

Figure S-1, which is based on the above relationship, shows for example an MTBF estimate of 58 hours in operational use for an average AGREE result of 100 hours. However, since the available data was not sufficient to develop relationships by classes of equipments, this is an <u>average</u> figure for all types of equipments and may not hold for specific classes of equipments.

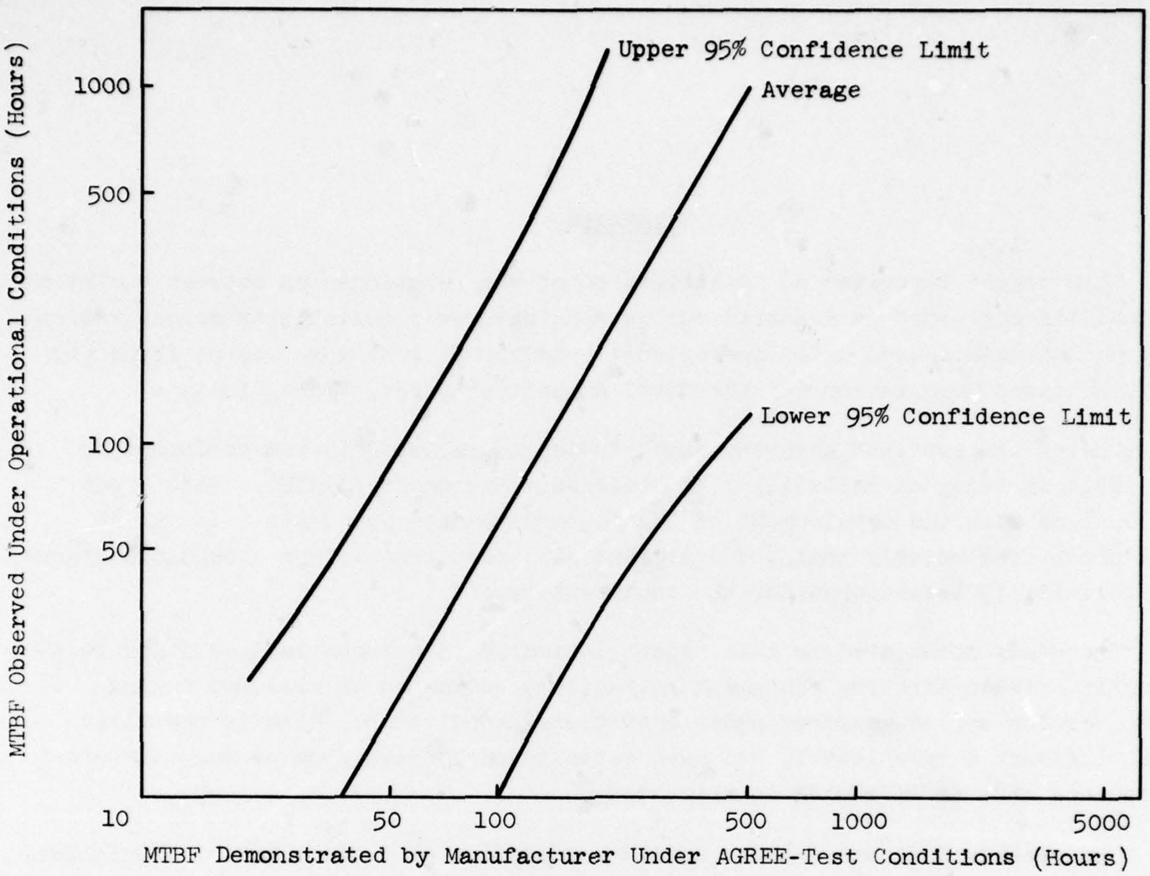

FIGURE S-1

RELATIONSHIP OF MTBF VALUES UNDER AGREE-TEST AND OPERATIONAL CONDITIONS

Multiple regression analysis was the basic statistical technique used in this study. Several factors other than the AGREE test results were recognized as pertinent to the accuracy of estimating expected field operational results. However, the data for all these factors were not available for each equipment used in the study. Consequently only the following additional factors were considered:

- Complexity of equipment (AEG - Active Element Group Count)
- Average mission length of aircraft
- Data collection methods
- Number of failures observed during operational use of the equipment

CONTENTS

1. INTRODUCTION

In recent years, dialogue on reliability testing has generated two distinct standpoints whose divergence concerns not so much the basic principle of reliability testing as what precisely is being measured. From one standpoint, a figure of merit in terms of freedom from catastrophic malfunction will suffice. From the other standpoint, it is considered essential to have a realistic assessment of actual field experience; effects such as the interaction among parts that comprise a system, and environmental factors such as imperfect support, maintenance, and operation must be accounted for.

In essence, the two approaches provide respectively a manufacturer's view and a user's view of reliability. Both types of assessment have their use in the overall drive toward attaining better system value. One of the earliest comprehensive studies to recognize these differences was performed by the Advisory Group for Reliability of Electronic Equipment (AGREE). In the widely read report[1] published by this group, nine task groups discussed and presented recommendations on the specific aspects of a reliability program.

The results reported by Task Group 3 are particularly relevant to this report. This Group developed the basic requirements for the AGREE tests; these were intended to serve as the guidelines which could be used to assure achievement of the reliabilities considered acceptable for given equipments. The specification of the various environmental levels for reliability testing in the AGREE publication is an indication of the effort to minimize the differences between laboratory test results and field results.

As stated almost ten years ago .... "the reliability index obtained from the equipment under test will be a useful measure of the field reliability even though the test conditions only approximately simulate the combined environmental effects which may be obtained in field use. Thus, while the measured MTBF may differ somewhat from that prevailing during operational use, this discrepancy will be small in contrast to that due to errors of measuring technique, differences in application, field maintenance, etc., ..." Equally important is the statement on the same page of the report to the effect that the tests are designed to measure the <u>inherent reliability</u> of the equipment under test. From the above statements it can be concluded that AGREE test results should approximate field results <u>only</u> if there are no errors in measuring technique (in the

[1] Advisory Group for Reliability of Electronic Equipments, Office of Assistant Secretary of Defense (R&D), June 1957.

field) or differences in application, field maintenance (due to personnel, training, procedures, logistics or test equipment, and similar factors.) However, it is essential that the user be assured that, when an equipment is used in the field, it will be at least as reliable as has been indicated during demonstration testing. To this end, this report documents a study to correlate AGREE laboratory results with field usage results and to establish the mathematical relationship between the two sets of results.

Section 2 discusses the sources and acquisition of data used in this study. Section 3 discusses the analysis of the data and the results of the analysis. Detailed discussion of various points and derivations of formulas are presented in the appendixes.

2. DATA SOURCES AND ACQUISITION

The scope of this study allowed only for the use of available data; no provisions were made for generating or collecting field data. The data sources were to be Navy material and ARINC Research Corporation's in-house references material.

The data used were restricted to those from airborne equipments. Although AGREE type data were located on 39 equipments, field operational data in sufficient quantity to yield meaningful MTBF values were available for only 20 of them -- and of these, the actual number of failures observed were available for only eleven. A summary of the data used is presented in Appendix A.

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3. ANALYSES AND DISCUSSION OF RESULTS

3.1 Mathematical Model

The relationships between field operational reliability and reliability under AGREE-test conditions, system parameters, and use factors were investigated by the statistical technique of multiple regression analysis. (See Appendix B for a discussion of this technique.) Regression was required because of the need to consider the effects of many variables simultaneously. The linear model was considered to be of sufficient accuracy for the study. The basic model of field usage reliability used for the regression analysis is:

$$Y_F = B_0 + B_1Y_A + B_2X_2 + \dots + B_nX_n$$

where

$X_2, X_3, \dots, X_n$ are system and use characteristics

$B_2, B_3, \dots, B_n$ are the true regression coefficients relating these characteristics to reliability

Y_F and Y_A are measures of field usage and AGREE reliability

B_1 is the true regression coefficient relating the measure of AGREE reliability to field usage reliability.

In this study, $\ell n\theta$* was used as the measure of reliability. Therefore, the basic regression equation became:

$$\ell n\theta_F = b_o + b_1(\ell n\theta_A) + b_2X_2 + \dots + b_nX_n$$

where

b_1 is a statistical estimate of B_1 and

θ is the mean time between failures of an equipment

In the analysis, $\ell n\theta_F$, $\ell n\theta_A$, $X_2X_3 \dots X_n$ are known for each equipment, and the regression analysis is essentially the solution of the set of such equations for the best values of the coefficients. For a prediction of a new equipment, the

*ℓn = Natural log

values of $\ln \theta_A$ X_2, . . . X_n are substituted and the values for b_o, b_1, . . . b_n determined above are used to compute the value of $\ln\theta_F$ where θ_F is the expected field mean time between failures.

3.2 Discussion of Results

Several factors other than the AGREE test results were recognized as pertinent to the accuracy of estimating expected field operational results. These included descriptions of equipment, environment, mission, and application; data collection methods; and reliability measures. However, the data for all these factors were not available for each equipment used in the study. Consequently, only the following additional factors were considered:

- Complexity of equipment (AEG)
- Average mission length of aircraft
- Data collection methods
- Number of failures observed during operational use of the equipment

3.2.1 Equipment Description

The only equipment description used in the regression runs was complexity as measured by Active Element Group (AEG) count. In most instances this factor was not significant. That is, no reduction in the variability in the estimates of operational reliability was obtained by including this factor with the AGREE test results.

3.2.2. Environmental Descriptors

Since all the AGREE tests were run at the same test level, (all equipments in the study were airborne equipments) the environmental variation was in too narrow a range to be used as a factor. Environmental data was not available for the operational data.

3.2.3 Mission and Application Descriptors

When the final relationship was derived, estimates for equipments that were used on more than one aircraft were used individually -- the average flight length of the aircraft was used to distinguish one estimate from another.

3.2.4 Influence of Data Source

The sample size was not sufficient to establish positively the degree to which the regression results were dependent on data source. However, 18 equipments' data, differently grouped according to source, were used to make four regression runs. The results are shown in Table 1; the specific data used for each run are identified in Table 3, Appendix A. Briefly, the first two runs used data from all 18 equipments and differed only in respect to the operational

TABLE 1

REGRESSION STATISTICS GENERATED WITH DIFFERENT DATA SUBSETS

Run Number	Source of Data Sets		Number of Equipments Represented	Correlation Coefficient	Standard Error of Estimate	Relationship
	Operational	AGREE				
1	ARINC and NATSF	Mfr and NATSF	18	0.73	0.75	$\theta_o = 0.81\theta_A^{0.93}$
2	ARINC and NATSF	Mfr and NATSF	18	0.73	0.80	$\theta_o = 0.64\theta_A^{0.98}$
3	ARINC	Mfr	8	0.69	0.80	$\theta_o = 0.92\theta_A^{1.24}$
4	NATSF	NATSF	6	0.86	0.77	$\theta_o = 0.09\theta_A^{1.32}$

MTBF values used for four equipments for which such data were available from two sources. The differences in the data sets did not appear to affect the regression statistics significantly. Runs 3 and 4 represented two groups of equipments distinguished by their different sources of data for both operational MTBF and AGREE-test MTBF. Although the regression coefficients are significantly different in these cases, it must be remembered that different equipments are represented in these runs.

3.2.5 Reliability Measures

The data that could be assembled within the scope of the program were not sufficiently detailed on a sufficient number of equipments to warrant performing regression runs on reliability measurements other than MTBF (e.g., total failures, relevant failures, complaints).

3.2.6 Final Relationship

For eleven of the equipments, data were available on the actual number of failures observed during operational use. This information provided valuable weighting factors with which to increase the accuracy of the regression results*. The run identified in Table 1 as Run 1 was repeated with the data appropriately weighted; the following regression statistics were generated:

Relationship: $\theta_o = 0.017\ \theta_A^{1.76}$

Correlation Coefficient: $r = 0.81$

Standard Error of Estimate: $\sigma = 0.70$

*(See Section 3.3 for discussion of the need for transforming the dependent variable and for using weighting factors.)

The correlation coefficient is a measure which shows what proportion of the original variation observed by a variable (operational MTBF) can be explained by the independent variable (AGREE-test MTBF). The standard error of estimate is a measure of the accuracy with which estimates may be made for new observations. The relationship represents equipments with a range of MTBF's obtained during operational use.

Figure 1 is derived from these final statistics and indicates that, on the average, equipments whose MTBF's are less than 200 hours under AGREE-test conditions will exhibit operational MTBF's that are lower than the AGREE results. However, due to the small sample size used in developing the relationship, this conjecture still requires verification.

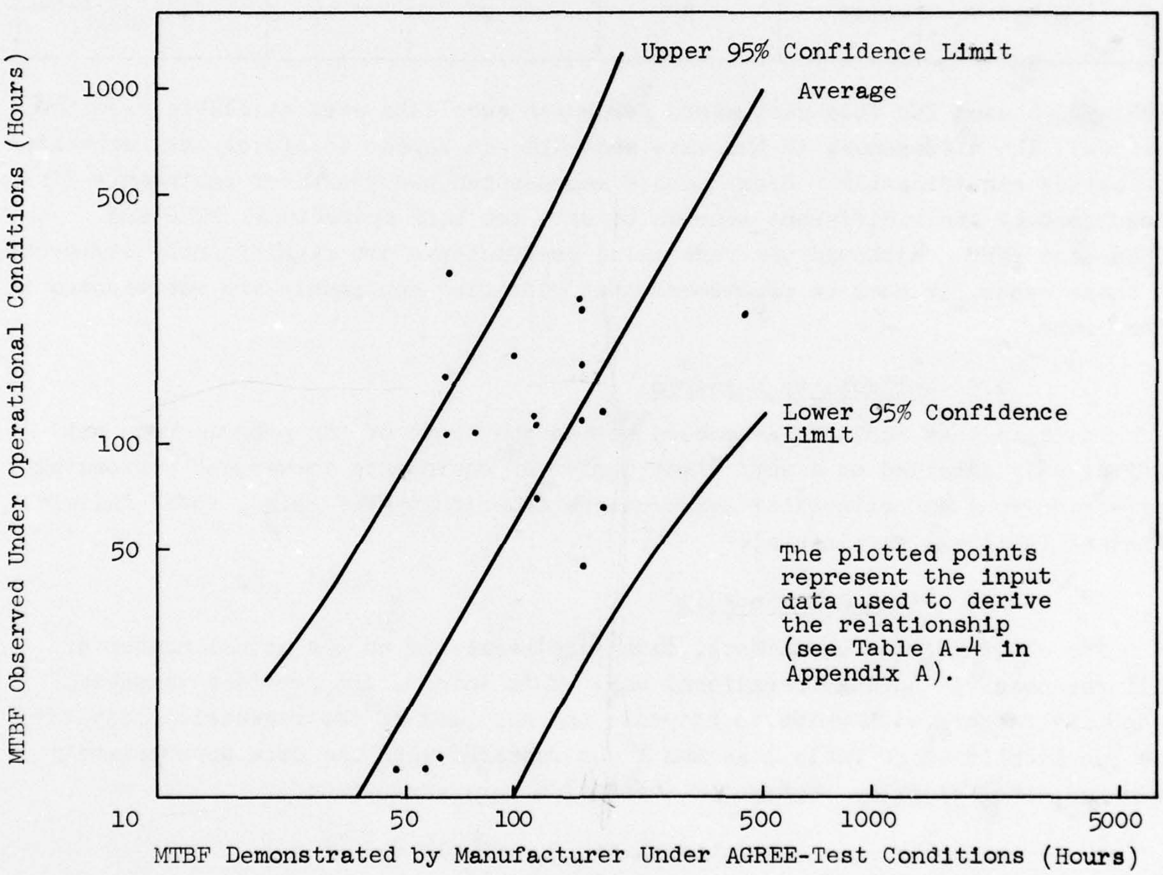

FIGURE 1

RELATIONSHIP OF MTBF VALUES UNDER AGREE-TEST AND OPERATIONAL CONDITIONS

This trend, illustrated by the average line in the figure, is probably associated with the fact that equipments that exhibit high reliability at the time of manufacture are less likely to be degraded in the field by maintenance-induced failures, whereas equipments that start with low reliability fail more often in the field and, therefore, are more susceptible to maintenance-induced failures.

If average flight length and AEG count are used in conjunction with the AGREE-test data to estimate field reliability, the following relationship -- which generated an R of 0.9 and a σ of 0.55 -- should be used:

$$\ell n\ \theta_O = 2.43 + 1.02\ \ell n\ \theta_A + 0.36\ (\text{flight length in hours}) - 0.75\ \ell n\ (\text{AEG's})$$

The use of this relationship in place of that plotted on Figure 1 results in a more accurate estimate. For example, a 100 hours MTBF demonstrated in the AGREE-type test resulted in an estimated operational MTBF of 58 hours. Use of the above relationship for various given flight lengths and AEG counts would result in the following estimates of operational MTBF:

Agree-Test MTBF	Flight Length (Hours)	AEG Count	Estimated Operational MTBF
100	2	100	87
		300	48
		500	24
	4	100	180
		300	100
		500	81
	5	100	258
		300	142
		500	72
300	2	100	268
		300	147
		500	75
	4	100	550
		300	304
		500	153
	5	100	785
		300	435
		500	219

The addition of flight-hours and AEG-count information provides a more accurate estimate of whether the operational MTBF will be less than or greater than the MTBF observed during the AGREE-type test. The future inclusion of additional significant factors into the analysis should enable even more precise estimation of expected operational reliability.

4. CONCLUSIONS AND RECOMMENDATIONS

This study has shown that there is a correlation between AGREE test results and operational reliability, although the developed relationship is based on a relatively small number of equipments all of which were airborne types. However, there will be considerably more operational data available in the near future (See Table A-1, Appendix A), and there is an indication that operational data covering Air Force equipments that have been subjected recently to AGREE testing will also be available soon.

It is recommended that additional regression analysis be performed as soon as field results are available on equipments that have recently been subjected to AGREE testing. This will provide a larger data base for deriving relationships of different classes of equipments and covering different test levels.

It would also be advisable to correlate the AGREE and operational test results to the various predictions made on the equipments to enable estimates of the operational reliability to be made as accurately and as early as possible.

APPENDIX A

DATA SOURCES AND SUMMARIES

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APPENDIX A

DATA SOURCES AND SUMMARIES

Equipments that have been subjected to AGREE testing were identified by reference to the following sources:

(1) Naval Air Systems Command, Code AIR-533E1

(2) Naval Air Technical Services Facility

(3) In-house ARINC Research sources

Table A-1 lists the equipments that these sources identify as having been subjected to AGREE testing. AGREE test level 3 was used on the airborne equipments for which field data were available. The data obtained from the Naval Air Systems Command consisted of the manufacturer's reports, except for the data on the AN/ARA-50, LN-14 and CADC which consisted only of a reported MTBF. There was a total of 39 equipments for which the results of AGREE testing were available.

Data from field operations were available at this time for only 20 of the 39 equipments that have been subjected to AGREE testing. The 20 equipments and the aircraft in which they were observed are identified in Table A-2. The mean times between failures for these equipments are presented in Table A-3. When data were available from more than one aircraft type, the values in the "Operational" columns represent averages for the aircraft types observed.

Table A-4 lists the data used in deriving the relationship

$$\theta_o = 0.017\theta_A^{1.76}.$$

Data from only eleven of the twenty equipments were used because the actual numbers of failures observed in the field (statistics necessary for an accurate derivation) were not available for the other nine equipments.

TABLE A-1

EQUIPMENTS SUBJECTED TO AGREE TESTING

Equipment Type	Source of AGREE-Test Data*	Availability of Operational Data	
		X = Available	Source*
Communications			
AN/ARC-34	c	X	c
AN/ARC-51	a,b		
AN/ARC-94	a,b	X	b
AN/ARC-21	c	X	c
AN/ARC-58	c	X	c
AN/ARC-102	a		
AN/ARC-104	a		
AN/ARR-66	a		
AN/ART-36	a		
AN/ARR-69	a		
AN/PRC-49	a		
AN/PRC-63	a		
AN/ARA-50	a		
AN/PRT-5	a		
AN/PRT-6	a		
Computer Sets			
AN/AJB-3A	a,b	X	b
AN/AYK-2	b	X	b
CADC	a	X	a
CNI Systems			
AN/ASQ-19	a	X	b,c
AN/ASQ-57	a	X	b,c
AN/ASQ-58	a	X	b,c
AN/ASQ-88	a		
Data Link Sets			
AN/ARR-60	c	X	c
AN/ARR-61	c	X	c
AN/USC-2 (Type 2)	b	X	b
AN/ASW-21	a		
Navigational Sets			
AN/ARN-52	a,b	X	b
AN/ASN-42	a,b	X	b,c
AN/ASN-50	a,b		
LN-14	a	X	a
Radar, Doppler			
AN/APN-102	c	X	c
AN/APN-141	a,b	X	b
AN/APN-153	b	X	b
AN/APN-167	a		
IFF			
AN/APX-46	a		
AN/APX-64	a		
Other			
AN/APR-27	a		
AN/ASH-50	a		
AN/ASM-198	a		

*(a) Naval Air Systems Command, Code AIR-533E1
(b) Naval Air Technical Services Facility
(c) In-House ARINC Research Sources

TABLE A-2

AIRCRAFT/EQUIPMENT COMBINATIONS FOR WHICH OPERATIONAL DATA ARE AVAILABLE

Equipment	P-3A	F-4B	E-2A	SH-3A	B-36	A-4E	KC-135	A-6A	B-52	VC-137	S-2E	F-102	F-111	F-100	F-101	B-47
AN/ASN-42	X															
AN/ASQ-19		X														
AN/ASQ-57								X								
AN/ASQ-58			X													
AN/AYK-2				X												
AN/AJB-3A		X														
AN/APN-141		X				X										
AN/APN-153	X		X			X		X								
AN/ARC-94	X					X		X			X					
AN/ARN-52	X			X							X					
AN/USC-2 (Type 2)			X													
AN/ARR-60															X	
AN/ARR-61															X	
AN/ARC-34														X		
AN/APN-102										X			X	X		X
AN/ARC-21					X				X							X
AN/ARC-58							X		X							
AN/ARA-50													X			
LN-14													X			
CADC													X			

TABLE A-3

MTBF VALUES UNDER AGREE-TEST AND OPERATIONAL CONDITIONS

Equipment Type	MTBF (Hours) for Conditions and Sources Shown			
	Under Operational Conditions		Under AGREE-Test Conditions	
	Source: ARINC Research	Source: NATSF	Source: Manufacturer	Source: NATSF
AN/ASN-42	85 (1,3)	107 (2,4)	78 (1,2,3)	61 (4)
AN/ASQ-19	9 (1,3)	12 (2)	47 (1,2,3)	
AN/ASQ-57	9 (1,3)	13 (2)	66 (1,2,3)	
AN/ASQ-58	13 (1,3)	12 (2)	56 (1,2,3)	
AN/AYK-2		228 (1,2)	454 (1,2)	
AN/AJB-3A		93 (1,2)	175 (1,2)	
AN/APN-141		109 (1,2,4)		115 (1,2,4)
AN/APN-153		181 (1,2,4)		155 (1,2,4)
AN/ARC-94		115 (1,2,4)		154 (1,2,4)
AN/ARN-52		140 (1,2,4)		67 (1,2,4)
AN/USC-2 (Type 2)		178 (1,2,4)		100 (1,2,4)
AN/ARR-60	74 (1,2,3)		164 (1,2,3)	
AN/ARR-61	113 (1,2,3)		198 (1,2,3)	
AN/ARC-34	33 (1,2,3)		41 (1,2,3)	
AN/APX-102	21 (1,2,3)		32 (1,2,3)	
AN/ARC-21	135		108*	
AN/ARC-58	292		79*	
AN/ARA-50		400 (1,2)	1,000 (1,2)	
LN-14		26 (1,2)	151 (1,2)	
CADC		91 (1,2)	126 (1,2)	

*Demonstrated at ambient conditions.

Note: The numbers in parentheses identify the regression runs for which the data were used (see Table 1 in the main text).

TABLE A-4

RELIABILITY DATA USED IN DERIVING THE MTBF RELATIONSHIPS

Equipment Type	AGREE-Test MTBF	Operational MTBF	Number of AEG's*	Average Flight Length	Number of Failures Observed
AN/ASN-42	78	107	530	5.7	510
AN/ASQ-19	47	12	325	1.7	479
AN/ASQ-57	66	13	391	1.9	536
AN/ASQ-58	56	12	447	2.1	262
AN/AYK-2	454	228	77	2.7	4
AN/AJB-3A	175	66	113	1.7	647
	175	121	113	1.6	605
AN/APN-153	155	235	253	5.7	161
	155	167	253	2.1	10
	155	256	253	1.6	91
	155	45	253	1.9	121
AN/ARC-94	154	115	188	5.7	478
	154	235	188	3.1	64
AN/ARN-52	67	301	144	5.7	154
	67	105	144	2.7	144
	67	151	144	3.1	188
AN/USC-2	100	178	349	2.1	19
AN/APN-141	115	111	200	1.7	658
	115	70	200	1.9	60
	115	120	200	1.7	47

*As reported in NATSF-MR No. 2, January through June 1965.

APPENDIX B

REGRESSION ANALYSIS

Adapted from

"System Reliability Prediction by Function"
Volume I, ARINC Research Corporation
Publication 241-01-1-375, 27 May 1963

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APPENDIX B

REGRESSION ANALYSIS

1. General Description

This Appendix summarizes the important characteristics of the multiple regression technique used for developing a prediction procedure. Some familiarity with basic statistical theory is assumed. Since this summary is necessarily limited in scope, certain sources* should be consulted for more detailed discussion of regression analysis.

The application of the regression technique presupposes some relationship between a dependent variable Y and one or more independent variables X_1, X_2, ..., X_r. The simplest case to consider is one in which the relationship can be approximated by a general linear equation of the form

$$Y = \beta_o + \sum_{i=1}^{r} \beta_i X_i + e \qquad \text{(B-1)}$$

The $\{\beta_i\}$ are parameters which, in n-dimensional space, generate the regression plane. The quantity e represents an error term, which is the measure of the variation in Y not accounted for by the regression plane. To achieve the general form of the equation, transformations may be applied to original Y and X values. For example, if $Y = A + B/X_1 + CX_2^2$, then the transformations $X_1' = 1/X_1$ and $X_2' = X_2^2$ yield an equation equivalent in form to equation (B-1).

Through analysis of data involving m observations of Y values and corresponding X values, e.g.,

Obs. 1: $Y_1, X_{11}, X_{21}, \ldots, X_{r1}$

Obs. 2: $Y_2, Y_{12}, X_{22}, \ldots, X_{r2}$

Obs. m: $Y_m, X_{1m}, X_{2m}, \ldots, X_{rm}$,

* Several good references are:
R. L. Anderson and T. A. Bancroft, Statistical Theory in Research, McGraw-Hill;
A. Hald, Statistical Theory with Engineering Applications, John Wiley & Sons:
C. Goulden, Methods of Statistical Analysis, John Wiley & Sons.

estimates of the $\{\beta_i\}$ can be obtained. This results in the estimating equation

$$Y = b_o + \sum_{i=1}^{r} b_i X_i + e \tag{B-2}$$

where the

$\{b_i\}$, termed regression coefficients, are estimates of the true but unknown $\{\beta_i\}$, and

e is the residual of the true Y about the estimated regression plane;

$$e = Y - (bo + \sum_{i=1}^{r} b_i X_i).$$

In the usual development of regression theory, the following assumptions and conditions are imposed:

(a) For estimating the regression equation, no assumptions of the distribution of the $\{X_i\}$ need be made. For inferential purposes, either of two general models may be involved:

Type I Model - The X's are fixed variables in that no probability distributions are associated with them.

Type II Model- The X's are considered to be stochastic variables.

(b) For a fixed set of X's, $\{X_i'\}$ the Y's are normally and independently distributed with mean $(\beta_o + \epsilon\ \beta_i X_i')$ and variance σ^2. (The normality assumption is only required for valid applications of the usual significance tests and confidence-interval estimation procedures. Non-extreme departures from normality are generally not too serious.)

(c) For every set of X's, the variance of Y is the same. (This assumption can be relaxed to include the case where the variance is proportional to the X's. See Section 3.3).

Assumptions (b) and (c) are equivalent to the assumption that the errors from the true regression surface are normally and independently distributed with zero mean and variance σ^2. From equation (4-2), the estimating equation for the expected value of Y is, therefore,

$$E\ (Y) = \hat{Y} = b_o + \sum_{i=1}^{r} b_i X_i. \tag{B-3}$$

With the above assumptions, it can be shown that the method of least squares is best for obtaining the estimates $\{b_i\}$ in the sense that these estimates are unbiased and have minimum variance. The least-squares method is one for which

the estimates $\{b_i\}$ are chosen to minimize the sum of squares of deviations from the estimated regression plane, termed the error sum of squares and defined mathematically by

$$SSE = \sum_{j=1}^{m} e_j^2$$

$$= \sum_{j=1}^{m} \left[Y_j - (b_o + \sum_{i=1}^{r} b_i X_{ij}) \right]^2 \qquad \text{(B-4)}$$

where Y_j is the j^{th} observed value of Y with corresponding X values of $\{X_{ij}\}$.

On applying the usual minimization procedures to equation (4-4) by differentiating with respect to the $\{b_i\}$, the well-known r "normal equations" are obtained, the k^{th} of which (k = 1, 2, ..., r) is

$$b_1 \epsilon x_k x_1 + b_2 \epsilon x_k x_2 + \ldots + b_k \epsilon x_k^2 + \ldots + b_r \epsilon x_k x_r = \epsilon x_k y \qquad \text{(B-5)}$$

where the summations are over the sample values and $x_i = X_i - \overline{X}_i$, $y = Y - \overline{Y}$, the deviation of a sample value from its sample mean.

Applying elementary algebraic methods to these r linear equations yields the least-squares estimates $\{b_i\}$.

2. Regression and Correlation Measures

The regression coefficients $\{b_i\}$ form the equation for predicting the expected value of Y for a given set of X's. The data used to obtain these estimates will also provide further information on the true relationship and characteristics of the prediction equation through calculation of various regression and correlation measures. Several of the more important measures are discussed in this section.

2.1 Variation About The Regression Plane

The scatter in the vertical or Y direction of the observed values of Y about the regression plane is perhaps the most important measure for evaluating the prediction ability of the derived equation. The population measure is the variance of e, denoted by σ^2. Provided the assumptions stated previously hold, an unbiased estimate of σ^2, e.g., s^2, can be obtained from the deviations of the observed Y values from the computed regression plane. The positive square root of this variance, s, is often called the standard error of estimate.

With the normality assumption and a large sample size, approximately 95% of the sample points will lie within ±2s from the estimated regression plane. Description of the population distribution in terms of confidence intervals is discussed in Section 2.5.

2.2 Variance of the Regression Coefficients

The variance of the $\{b_i\}$ are also obtainable from the source data. These statistics are useful for comparing regression coefficients for two sets of data, for evaluating the significance of a variable on the overall regression, and for determining if an observed b differs significantly from a theoretical value.

2.3 Correlation Measures

Correlation theory is concerned primarily with measuring the degree of association or covariation between two or more variables -- as differentiated from regression theory, which attempts to describe the relationship through a regression equation. Since correlation theory has been fully developed only when Y and the X's form a multivariate normal distribution, interpretation of correlation measures for other cases is somewhat uncertain. However, correlation measures can be used for significance tests (see Section 2.4) and do offer some idea of the degree of association, provided that the values of the independent variables are not preselected.

2.3.1 Simple Correlation Coefficient

Given two variables X and Y, a simple correlation coefficient is defined by

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

where σ_x and σ_y are standard deviations of X and Y respectively and σ_{xy} is the covariance between X and Y defined as $E\{[X-E(X)][Y-E(Y)]\}$. It can be shown that $-1 \leq \rho \leq 1$.

A more meaningful description of the correlation coefficient can be presented through a regression viewpoint. Let σ^2_y represent the variance of Y in terms of deviations from the mean of all Y values. Let $\sigma^2_{y|x}$ represent the variance of Y when deviations from the regression line of Y and X are used. Then $(\sigma^2_y - \sigma^2_{y|x})$ represents the reduction in the variance of Y due to estimating Y from X rather than from the average Y value. The square root of the fraction of total variation accounted for by regression is the population correlation coefficient, i.e.,

$$\sqrt{\rho = \frac{\sigma^2 y - \sigma^2 y|x}{\sigma^2_y}} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

If $\rho = 0$, then $\sigma^2_y = \sigma^2_{y|x}$, implying no reduction in σ^2_y through knowledge of X; in other words, the regression line has slope 0 and is coincident with the mean of Y. If $\rho = \pm 1$, then $\sigma^2_{y|x} = 0$, implying all points (X,Y) lie on the regression line. The sign of ρ is the same as the slope of the regression line.

2.3.2 Multiple Correlation Coefficient

The multiple correlation coefficient, R, is a relative measure of the association between three or more variables. It is analogous to the simple correlation coefficient except that its sign is always taken to be positive. The square of the multiple correlation coefficient, is known as the coefficient of determination. This quantity can be used to determine if the addition of an independent variable is significant by comparison of the sample R^2 for (n-1) variables to the sample R^2 for n variables.

2.3.3 Partial Correlation Coefficient

A partial correlation coefficient measures the association between two variables when the influence of all other variables considered is eliminated. Simple correlation merely ignores the influence of other variables. Partial correlation coefficients give insight into which independent variables are closely related to the dependent variable.

2.4 Significance Tests

With the assumption of normally and independently distributed Y values for a given set of $\{X_i\}$, various tests may be performed to determine the statistical significance of the computed statistics.

The overall significance of the regression can be determined through an F test by comparison of the sum of squares due to regression with the sum of squares due to regression with the sum of squares due to error. We have

$$SSE = \sum_{j=1}^{m} (y_j - \sum_{i=1}^{r} b_i x_{ij})^2$$

where the y and $\{x_i\}$ are deviations from their respective means. On expanding the summation we find

$$SSE = \sum_{j=1}^{m} y_j^2 - \sum_{i=1}^{r} b_i \sum_{j=1}^{m} x_{ij} y_j \qquad \text{(B-6)}$$

Since ϵy_j^2 represents the total sum of squares, the second term on the right side of equation B-6 represents the sum of squares due to regression, or

$$SSR = \sum_{i=1} b_i \sum_{j=1} x_{ij} Y_j . \qquad \text{(B-7)}$$

It can be shown that if all β_i are equal to zero, the case equivalent to no regression, then

$$E(SSE) = (m-r-1)\ \sigma^2$$

$$E(SSR) = r\ \sigma^2$$

To test this hypothesis, compute the quantity (SSR) (m-r-1)/(SSE) (r) which is distributed as F with r and (m-r-1) degrees of freedom. The associated analysis-of-variance table is given below.

ANALYSIS-OF-VARIANCE TABLE FOR TESTING SIGNIFICANCE OF THE OVERALL REGRESSION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Regression (r independent Variables)	r	SSR	SSR/r
Error	m-r-1	SSE	$\frac{SSE}{m-r-1} = s^2$
Total	m-1	$\sum_{j=1}^{m} y_j^2$	

Division of SSR and SSE by Σy^2 yields the statistic

$$F = \frac{(SSR)\ (n-r-1)/\Sigma_y^{\ 2}}{(SSE)(r)/\Sigma_y^{\ 2}} = \frac{R^2(m-r-1)}{(1-R^2)(r)} \qquad \text{(B-8)}$$

Which is often used for the significance test.

In a similar manner, the addition of (r - p) independent variables to the regression equation is tested for significance by computing the statistic

$$F = \frac{(SSR_r - SSR_p)\ (m-r-1)}{(SSE)\ (r-p)}$$

$$= \frac{(R_r^{\ 2} - R_p^{\ 2}\ (m-r-1)}{(1-R^2_{\ r})\ (r-p)} \qquad \text{(B-9)}$$

where the subscripts r and p refer to the number of independent variables being considered.

Tests on individual variables may be performed by examining the hypothesis that a particular $\beta_i = 0$. This is accomplished through use of the fact that the

quantity $(b_1 - \beta_1)/s_{b1}$, where $s_{b1}^2/\sum_{j=1}^{m} x_j^2$, is distributed as Student's t with (m-r-1) degrees of freedom.

2.5 Confidence Interval Estimates

Equation B-3 provides a point estimate of the expected value of Y for a known set of X's. Confidence limits for the true mean value of Y as well as limits for an individual Y value for a given set of X's are also obtainable from the data through the assumption of the normality of the distribution of the Y arrays.

For the confidence interval on the mean value of Y, the variance of $[\hat{Y} - E(Y)]$ is required. For just one independent variable that is assumed to have the value X', with a corresponding predicted value of the mean of Y equal to $\hat{Y}'$,

$$\hat{Y}' - E(Y) = b_o + b_1 X' - (\beta_o + \beta_o X')$$

$$= (b_o - \beta_o) + (b_1 - \beta_1) X'$$

and

$$\mathrm{var}\,[\hat{Y}' - E(Y)] = \sigma^2 \left[\frac{1}{m} + \frac{(X' - \bar{X})^2}{\sum_{j=1}^{m} (X_j - \bar{X})^2} \right] \qquad \text{(B-10)}$$

For an individual Y value,

$$\mathrm{var}\,(\hat{Y}' - Y) = \sigma^2 \left[1 + \frac{1}{m} + \frac{(X' - \bar{X})^2}{\sum_{j=1}^{m} (X_j - \bar{X})^2} \right] \qquad \text{(B-11)}$$

Using s^2, the estimate of σ^2, in equations 4-10 and 4-11, the 100 $(1 - \alpha)$% confidence interval for E(Y) is

$$\hat{Y}' \pm t_{\alpha/2} \sqrt{\mathrm{var}\,[\hat{Y}' - E(Y)]} \qquad \text{(B-12)}$$

The $(1 - \alpha)$% confidence interval for an individual Y value is

$$\hat{Y}' \pm t_{\alpha/2} \sqrt{\mathrm{var}\,(\hat{Y}' - Y)} \qquad \text{(B-13)}$$

Extensions of equations (4-12) and (4-13) can be obtained for the multivariate case; they are discussed in Section 4.3, where computational aspects are considered.

3. Application to Reliability Prediction

3.1 Type of Regression Model Relationships

As in many practical applications, strict conformance to the theoretical requirements was not possible for this project. The types of regression models appropriate to the variables used were not preselected, but were those already existing. Some of the variables, such as AEG counts, can be considered to be fixed with no measurement errors. Others, such as frequency and power consumption, do have associated probability distributions and possibly non-negligible measurement errors.

Therefore, a strict designation of a Type I or Type II model cannot be made. This, however, will not affect the analysis with respect to the estimate of the regression equation if the listed assumptions are reasonably satisfied. It is further believed that the degree of rigor achieved with respect to other aspects of the analysis is satisfactory and consistent with the intended use of the results.

3.2 Transformations of Independent Variables

As indicated previously, a model that is linear in the regression parameters is required. Transformations were applied to several of the independent variables to linearize suspected logarithmic or multiplicative relationships through consideration of the physical processes involved.

In those cases where no prior knowledge of judgments existed, transformations that gave the best results were used. The transformations were usually determined on the basis of analyzing the partial correlation coefficients, or the significance of the regression coefficients for two or more transformations that were throught to be reasonable for the variable under consideration.

Graphical procedures normally used to determine the form of the relationship, and thus the appropriate transformation, were not feasible because of the many independent variables involved.

3.3 Transformation of Dependent Variable

The dependent variable under consideration is the equipment mean life expressed as the mean time between failures (MTBF). When the failure times of the systems were approximately distributed in accordance with the exponential assumption, i.e., the density of failure time t, is

$$f(t) = \frac{1}{\theta} e^{-t/\theta} \tag{B-14}$$

where θ is the true mean life.

If k failures are observed in a total of T hours, the estimate of θ is

$$\hat{\theta} = \frac{T}{k} \tag{B-15}$$

The density of the statistic $\hat{\theta}$ can be shown to be

$$g(\hat{\theta}) = \frac{1}{(k-1)!} \left(\frac{k}{\theta}\right)^k \hat{\theta}^{k-1} e^{-k\hat{\theta}/\theta}$$

$$= \frac{1}{\Gamma(k)} \frac{1}{\left(\frac{\theta}{k}\right)^k} \hat{\theta}^{k-1} e^{-k\hat{\theta}/\theta} \tag{B-16}$$

Equation (B-16) represents a gamma density with a mean of θ and variance θ^2/k. Thus, even if the number of failures were identical for all systems, the variances of the Y's are not constant but are a function of the true mean lives of the system. A common procedure for stabilizing the variance when it is proportional to the square of the mean is to employ the logarithmic transformation $Y = \log \hat{\theta}$. This transformation, fortunately, will also tend to yield an approximately normal distribution for the Y's*.

In order to account for varying number of failures, the Y values were also weighted by their respective number of failures, since if var $(\hat{\theta}) = \theta^2/k$, then through use of a Taylor's series, var $(\log \hat{\theta}) \approx 1/k$. The normal equations are then obtained by minimizing

$$SSE = \sum_{j=1}^{m} w_j (Y_j - b_o - \sum_{i=1}^{r} b_i X_{ij})^2 \tag{B-17}$$

with respect to the $\{b_i\}$ where $Y_j = \log \hat{\theta}_j$ and w_j is proportional to the number on the j^{th} equipment type.

3.4 Computational Aspects

This section summarizes the formulas and approach used for obtaining the prediction equation, performing significance tests, and obtaining the final results.

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* See, for example, K. Brownlee, Statistical Theory and Methodology in Science and Engineering, John Wiley and Sons, p. 115.

3.4.1 Regression and Correlation Statistics

Notation:

n = total number of variables (also the subscript pertaining to the dependent variable)

$r = n-1$ = total number of independent variables

m = total number of sample observations (equivalent to the number of equipment types)

X_{ij} = value of the i^{th} variable on the j^{th} observation ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$)

S = summation operation over the sample observation;

$$\text{e.g., } SX_i = \sum_{j=1}^{m} X_{ij}.$$

Sample means

$$\overline{X}_i = \frac{SX_i}{m}. \tag{B-18}$$

Sample variances:

$$s_i^2 = \frac{S(X_i - \overline{X}_i)^2}{m-1} = \frac{m\, S\, X_i^2 - (SX_i)^2}{m\,(m-1)} \tag{B-19}$$

Simple Correlation Coefficients:

$$r_{ij} = \frac{S(X_i - \overline{X}_i)\,(X_j - \overline{X}_j)}{(m-1)\; s_i\; s_j}$$

$$= \frac{mSX_iX_j - SX_i\; SX_j}{(m-1)\; s_i\; s_j} \tag{B-20}$$

For obtaining multiple regression statistics by computer utilization, the inverse matrix of the simple correlation coefficients can be used advantageously. Let $r_{ij}^{-1} = a_{ij}$ represent an element of this inverse matrix. Then the following computational equations hold:

Regression Coefficients:

$$b_i = \frac{a_{ni}}{a_{nn}}\,\frac{s_n}{s_i} \quad i = 1, 2, \ldots, r \tag{B-21}$$

$$b_o = \overline{X}_n - \sum_{i=1}^{r} b_i\overline{X}_i. \tag{B-22}$$

Partial Correlation Coefficients:

$$r_{ni.} = -\frac{a_{ni}}{\sqrt{a_{nn}\, a_{ii}}} \qquad i = 1, 2, \ldots, r. \tag{B-23}$$

Standard Error of the Regression Coefficients:

$$s_{bi} = \frac{b_i}{r_{ni.}} \sqrt{\frac{1-r_{ni.}}{m-n}}, \quad i = 1, 2, \ldots, r. \tag{B-24}$$

Multiple Correlation Coefficient:

$$R = \sqrt{1 - \frac{1}{a_{nn}}}. \tag{B-25}$$

Standard Error of Estimate:

$$s = s_n \sqrt{\frac{m}{(m-n)a_{nn}}}. \tag{B-26}$$

3.4.2 Computational Procedure for Significance Tests

For a set of n variables, the overall significance of the regression can be evaluated by computing the following *statistic*

$$F = \frac{R^2(m-n)}{1-R^2(n-1)} \tag{B-27}$$

and comparing it with a critical F for (n-1) and (m-n) degrees of freedom at a preselected significance level of α. If the calculated F is greater than the critical F, then the hypothesis that all of the β_i are equal to zero is rejected and the overall regression is judged to be significant.

In order to determine which of the r independent variables are the significant contributors to the regression, a systematic procedure is required to reduce the computational problem to manageable size.

To illustrate the problem, if it were decided that, at most, five of twenty independent variables would be finally selected, one approach would be to try all possible combinations of five-out-of-twenty and pick the best of these. This would then involve at least $\binom{20}{5}$, or more than 15,000 regression runs, a task prohibitively expensive even for the larger electronic computers.

The systematic approach used in this project for significant variable selection is called the "square root" method; it is described in detail in a paper by A. Summerfield and A. Lubin:

> "A Square Root Method of Selecting a Minimum Set of Variables in Multiple Regression", <u>Psychometrika</u> Part I: The Method; Volume 16, No. 3, September 1951, pp. 271-284.
> Part II: A Worked Example; Volume 16, No. 4, December 1951, pp. 425-437.

This method involves determining partial correlations of each independent variable with the dependent variable when the influence of all previously selected variables is eliminated. The highest semi-partial correlation indicates which of the remaining non-selected variables will add most to the overall regression. The reduction in the error sum of squares (addition to R^2) attributed to this variable is then tested by the F-ratio criterion. If the variable is determined to be insignificant, no other single remaining variable will be significant.*

The computational procedure involves the construction of a triangular matrix (called the square root matrix by Summerfield and Lubin) of semi-partial correlation coefficients from the original matrix of simple correlation coefficients. If S_{ij} represents an element of this square matrix, it has the following properties:

$S_{ij} = 0$ for $i < j$

S_{ij} = semi-partial correlation of variable j to variable i when variables 1, 2, ..., j-1 (preselected) are held constant

$S_{i1} = r_{i1}$ for $i = 1, \ldots, n$.

The S_{ij} are obtained as follows:

$$S_{ii} = \left[1 - \sum_{j=1}^{i=1} (S_{ij})^2 \right]^{1/2}, \quad i = 1, 2, \ldots, n. \qquad \text{(B-28)}$$

$$S_{ij} = r_{ij} - \frac{\sum_{k=1}^{i-1} S_{ik}\, S_{jk}}{S_{ii}} \qquad i > j,\ i \geq 2,\ j = 1, 2, \ldots, r. \qquad \text{(B-29)}$$

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* It is possible to consider the addition of a combination of variables. This has not been done directly in this project.

The selection order is determined as follows:

($\underline{S}_{1a}(b)$ = the a^{th} column of the square root matrix when variable b is assumed to be the a^{th} most important:

(1) That variable j for which r_{jn} is a maximum is the first selected variable. Call this variable j_1. Then $\underline{S}_{i1} = \underline{r}_{ji1}$.

(2) Calculate S_{n2} (j) for all $j \neq j_1$. The value j for which $S_{na}(j)$ is a maximum is the second most important variable. Call this variable j_2. The second column of the square root matrix is then $\underline{S}_{i2} = \underline{S}_{i2}(j_2)$.

(3) In general, for obtaining the p^{th} most important variable after variables $j_1, j_2, \ldots, j_{p-1}$ have been selected, calculate S_{np} (j) for $j \neq j_1, j_2, \ldots, j_{p-1}$. That value j for which $S_{np}(j)$ is a maximum is selected and $\underline{S}_{np} = \underline{S}_{np}(j_p)$.

After each selection an F test is performed to determine if the addition to R^2 is significant. Thus, to determine if the p^{th} selected variable is significant, compute

$$F = \frac{(R^2{}_p - R^2{}_{p-1})\ (m-1-P)}{(1-R^2{}_p)} \tag{B-30}$$

and compare to a critical F based on 1 and (m-1-p) degrees of freedom for a chosen significance level α.

The final regression results can then be obtained from the correlation matrix and the means and variances of those variables determined to be significant.

3.4.3 Confidence Interval Prediction

To obtain estimates of the variances required for confidence interval prediction, namely var $[\hat{Y}-E(Y)]$ and var $(\hat{Y} - Y)$, the following equations are applicable:

$$\text{var}\ [\hat{Y}' - E(Y)] = s^2 \left[\frac{1}{m} + \sum_{i,j=1}^{r} c_{ij}\ (X_i' - \overline{X}_i)\ (X_j' - \overline{X}_j) \right] \tag{B-31}$$

$$\text{var}\ (\hat{Y}' - Y) = s^2 + \text{var}\ [\hat{Y}' - E(Y)] \tag{B-32}$$

where the c_{ij} are known as Gauss multipliers. They can be obtained from the inverse correlation matrix by the following equations:

$$c_{ii} = \frac{a_{nn}\ a_{ii} - a_{ni}{}^2}{(m-1)s^2{}_i\ a_{nn}} \tag{B-33}$$